ode45, ode23, ode113, ode15s, ode23s

Purpose

Solve differential equations

Syntax

```
[T,Y] = solver('F',tspan,y0)
[T,Y] = solver('F',tspan,y0,options)
[T,Y] = solver('F',tspan,y0,options,p1,p2...)
[T,Y,TE,YE,IE] = solver('F',tspan,y0,options)
[T,X,Y] = solver('model',tspan,y0,options,ut,p1,p2,...)
```

Arguments

`F`	Name of the ODE file, a MATLAB function of `t` and `y` returning a column vector. All solvers can solve systems of equations in the form $y' = F(t, y)$. `ode15s` and `ode23s` can both solve equations of the form $My' = F(t, y)$. Only `ode15s` can solve equations in the form $M(t)y' = F(t, y)$. For information about ODE file syntax, see the `odefile` reference page.
`tspan`	A vector specifying the interval of integration `[t0 tfinal]`. To obtain solutions at specific times (all increasing or all decreasing), use `tspan = [t0,t1, ..., tfinal]`.
`y0`	A vector of initial conditions.
`options`	Optional integration argument created using the `odeset` function. See `odeset` for details.
`p1,p2...`	Optional parameters to be passed to `F`.
`T,Y`	Solution matrix `Y`, where each row corresponds to a time returned in column vector `T`.

Description

`[T,Y] = solver('F',tspan,y0)` with `tspan = [t0 tfinal]` integrates the system of differential equations *y′ = F(t,y)* from time `t0` to `tfinal` with initial conditions `y0`. `'F'` is a string containing the

name of an ODE file. Function `F(t,y)` must return a column vector. Each row in solution array `y` corresponds to a time returned in column vector `t`. To obtain solutions at the specific times `t0`, `t1`, ..., `tfinal` (all increasing or all decreasing), use `tspan = [t0 t1 ... tfinal]`.

`[T,Y] = solver('F',tspan,y0,options)` solves as above with default integration parameters replaced by property values specified in `options`, an argument created with the `odeset` function (see `odeset` for details). Commonly used properties include a scalar relative error tolerance `RelTol` (`1e-3` by default) and a vector of absolute error tolerances `AbsTol` (all components `1e-6` by default).

`[T,Y] = solver('F',tspan,y0,options,p1,p2...)` solves as above, passing the additional parameters `p1,p2...` to the M-file `F`, whenever it is called. Use `options = []` as a place holder if no options are set.

`[T,Y,TE,YE,IE] = solver('F',tspan,y0,options)` with the `Events` property in `options` set to `'on'`, solves as above while also locating zero crossings of an event function defined in the ODE file. The ODE file must be coded so that `F(t,y,'events')` returns appropriate information. See `odefile` for details. Output `TE` is a column vector of times at which events occur, rows of `YE` are the corresponding solutions, and indices in vector `IE` specify which event occurred.

When called with no output arguments, the solvers call the default output function `odeplot` to plot the solution as it is computed. An alternate method is to set the `OutputFcn` property to `'odeplot'`. Set the `OutputFcn` property to `'odephas2'` or `'odephas3'` for two- orthree-dimensional phase plane plotting. See `odefile` for details.

For the stiff solvers `ode15s` and `ode23s`, the Jacobian matrix $\partial F/\partial y$ is critical to reliability and efficiency so there are special options. Set `JConstant` to `'on'` if $\partial F/\partial y$ is constant. Set `Vectorized` to `'on'` if the ODE file is coded so that `F(t,[y1 y2 ...])` returns `[F(t,y1) F(t,y2) ...]`. Set `Jattern` to `'on'` if $\partial F/\partial y$ is a sparse matrix and the ODE file is coded so that `F([],[],'jpattern')` returns a sparsity pattern matrix of `1`'s and `0`'s showing the nonzeros of $\partial F/\partial y$. Set `Jacobian` to `'on'` if the ODE file is coded so that `F(t,y,'jacobian')` returns $\partial F/\partial y$.

Both `ode15s` and `ode23s` can solve problems

$My' = F(t, y)$ with a constant mass matrix *M* that is nonsingular and (usually) sparse. Set `Mass` to `'on'` if the ODE file is coded so that `F([],[],'mass')` returns *M* (see `fem2ode`). Only `ode15s` can solve problems

$M(t)y' = F(t, y)$ with a time-dependent mass matrix *M(t)* that is nonsingular and (usually) sparse. Set `Mass` to `'on'` if the ODE file is coded so that `F(t,[],'mass')` returns $M(t)$ (see `fem1ode`). For `ode15s` set `MassConstant` to `'on'` if *M* is constant.

Solver	Problem Type	Order of Accuracy	When to Use
`ode45`	Nonstiff	Medium	Most of the time. This should be the first solver you try.
`ode23`	Nonstiff	Low	If using crude error tolerances or solving moderately stiff problems.
`ode113`	Nonstiff	Low to high	If using stringent error tolerances or solving a computationally intensive ODE file.
`ode15s`	Stiff	Low to medium	If `ode45` is slow (stiff systems) or there is a mass matrix.
`ode23s`	Stiff	Low	If using crude error tolerances to solve stiff systems or there is a constant mass matrix.

The algorithms used in the ODE solvers vary according to order of accuracy [5] and the type of systems (stiff or nonstiff) they are designed to solve. See Algorithms on page 2-473 for more details.

It is possible to specify `tspan`, `y0` and `options` in the ODE file (see `odefile`). If `tspan` or `y0` is empty, then the solver calls the ODE file:

```
[tspan,y0,options] = F([],[],'init')
```

to obtain any values not supplied in the solver's argument list. Empty arguments at the end of the call list may be omitted. This permits you to call the solvers with other syntaxes such as:

```
[T,Y] = solver('F')
[T,Y] = solver('F',[],y0)
[T,Y] = solver('F',tspan,[],options)
[T,Y] = solver('F',[],[],options)
```

Integration parameters (`options`) can be specified both in the ODE file and on the command line. If an option is specified in both places, the command line specification takes precedence. For information about constructing an ODE file, see the `odefile` reference page.

Options

Different solvers accept different parameters in the options list. For more information, see `odeset` and *Using MATLAB* .

Parameters	ode45	ode23	ode113	ode115s	ode23s
RelTol, AbsTol	√	√	√	√	√
OutputFcn, OutputSel, Refine, Stats	√	√	√	√	√
Events	√	√	√	√	√
MaxStep, InitialStep	√	√	√	√	√
JConstant, Jacobian, JPattern, Vectorized	--	--	--	√	√
Mass MassConstant	-- --	-- --	-- --	√ √	√ --
MaxOrder, BDF	--	--	--	√	--

Examples

Example 1. An example of a nonstiff system is the system of equations describing the motion of a rigid body without external forces:

$$
\begin{aligned}
y'_1 &= y_2 y_3 & y_1(0) &= 0 \\
y'_2 &= -y_1 y_3 & y_2(0) &= 1 \\
y'_3 &= -0.51 y_1 y_2 & y_3(0) &= 1
\end{aligned}
$$

To simulate this system, create a function M-file `rigid` containing the equations:

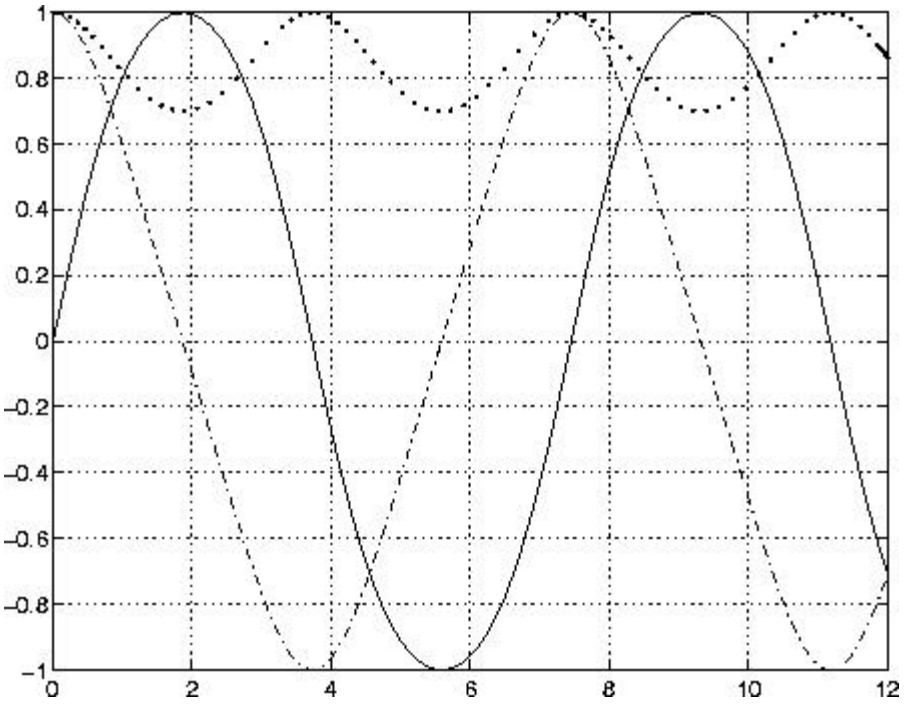
```
function dy = rigid(t,y)
dy = zeros(3,1);    % a column vector
dy(1) = y(2) * y(3);
dy(2) = -y(1) * y(3);
dy(3) = -0.51 * y(1) * y(2);
```

In this example we will change the error tolerances with the `odeset` command and solve on a time interval of [0 12] with initial condition vector [0 1 1] at time `0`.

```
options = odeset('RelTol',1e-4,'AbsTol',[1e-4 1e-4 1e-5]);
[t,y] = ode45('rigid',[0 12],[0 1 1],options);
```

Plotting the columns of the returned array `Y` versus `T` shows the solution:

```
plot(T,Y(:,1),'-',T,Y(:,2),'-.',T,Y(:,3),'.')
```



Example 2. An example of a stiff system is provided by the van der Pol equations governing relaxation oscillation. The limit cycle has portions where the solution components change slowly and the problem is quite stiff, alternating with regions of very sharp change where it is not stiff.

$$y'_1 = y_2 \qquad y_1(0) = 0$$
$$y'_2 = 1000(1 - y_1^2)y_2 - y_1 \qquad y_2(0) = 1$$

To simulate this system, create a function M-file `vdp1000` containing the equations:

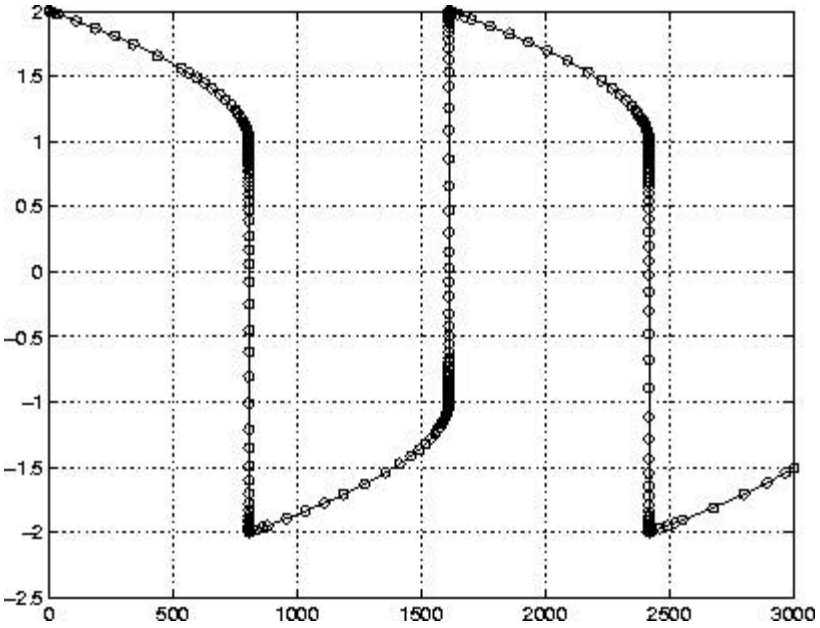
```
function dy = vdp1000(t,y)
dy = zeros(2,1);    % a column vector
dy(1) = y(2);
dy(2) = 1000*(1 - y(1)^2)*y(2) - y(1);
```

For this problem, we will use the default relative and absolute tolerances (`1e-3` and `1e-6`, respectively) and solve on a time interval of [0 3000] with initial condition vector [2 0] at time `0`.

```
[T,Y] = ode15s('vdp1000',[0 3000],[2 0]);
```

Plotting the first column of the returned matrix `Y` versus `T` shows the solution:

```
plot(T,Y(:,1),'-o'):
```



Algorithms

`ode45` is based on an explicit Runge-Kutta (4,5) formula, the Dormand-Prince pair. It is a *one-step* solver - in computing `y(tn)`, it needs only the solution at the immediately preceding time point, `y(tn-1)`. In general, `ode45` is the best function to apply as a "first try" for most problems. [1]

`ode23` is an implementation of an explicit Runge-Kutta (2,3) pair of Bogacki and Shampine. It may be more efficient than `ode45` at crude tolerances and in the presence of moderate stiffness. Like `ode45`, `ode23` is a one-step solver. [2]

`ode113` is a variable order Adams-Bashforth-Moulton PECE solver. It may be more efficient than `ode45` at stringent tolerances and when the ODE file function is particularly expensive to evaluate. `ode113` is a *multistep* solver - it normally needs the solutions at several preceding time points to compute the current solution. [3]

The above algorithms are intended to solve non-stiff systems. If they appear to be unduly slow, try using one of the stiff solvers (`ode15s` and `ode23s`) instead.

`ode15s` is a variable order solver based on the numerical differentiation formulas, NDFs. Optionally, it uses the backward differentiation formulas, BDFs (also known as Gear's method) that are usually less efficient. Like `ode113`, `ode15s` is a multistep solver. If you suspect that a problem is stiff or if `ode45` has failed or was very inefficient, try `ode15s`. [7]

`ode23s` is based on a modified Rosenbrock formula of order 2. Because it is a one-step solver, it may be more efficient than `ode15s` at crude tolerances. It can solve some kinds of stiff problems for which `ode15s` is not effective. [7]

See Also

`odeset`, `odeget`, `odefile`

References

[1] Dormand, J. R. and P. J. Prince, "A family of embedded Runge-Kutta formulae," J. Comp. Appl. Math., Vol. 6, 1980, pp 19-26.

[2] Bogacki, P. and L. F. Shampine, "A 3(2) pair of Runge-Kutta formulas," Appl. Math. Letters, Vol. 2, 1989, pp 1-9.

[3] Shampine, L. F. and M. K. Gordon, *Computer Solution of Ordinary Differential Equations: the Initial Value Problem*, W. H. Freeman, San Francisco, 1975.

[4] Forsythe, G. , M. Malcolm, and C. Moler, *Computer Methods for Mathematical Computations*, Prentice-Hall, New Jersey, 1977.

[5] Shampine, L. F. , *Numerical Solution of Ordinary Differential Equations*, Chapman & Hall, New York, 1994.

[6] Kahaner, D. , C. Moler, and S. Nash, *Numerical Methods and Software*, Prentice-Hall, New Jersey, 1989.

[7] Shampine, L. F. and M. W. Reichelt, "The MATLAB ODE Suite," (to appear in *SIAM Journal on Scientific Computing*, Vol. 18-1, 1997).

[Previous | Help Desk | Next]